

Late

Math

LOAN TABLES

BY

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The following Tables are an extension of a set published in the *Canadian Almanac* for 1870, which gave the values from 1 to 12 years for rates of interest per cent. per annum 10, $10\frac{1}{2}$, 11, $11\frac{1}{2}$, 12 payable half-yearly. In the present set, the time runs up to 20 years, and the rates are 8, 9, 10, 11, 12, 13; with some additional Tables which seemed likely to be of use.

EXPLANATION AND USE OF THE TABLES.

When a loan is contracted to be repaid (principal and interest) by equal instalments extending over a term of years, the following tables will give the amount of the instalment, monthly, quarterly, half-yearly, or yearly, for various rates of interest, and will also give the present worth or surrender value of the instalments for any unexpired portion of the term.

As interest is in this country generally payable half-yearly, the half-yearly rate has been taken as the basis, and the calculations are made on the true yearly rate corresponding and not the nominal rate. Thus 10 per cent. per annum, payable half-yearly, is understood to mean 5 per cent. per half-year, the true yearly rate in this case being $10\frac{1}{4}$; for, \$100 being put out for one year at 5 per cent. per half-year, would amount at the end of the year to \$110.25. Both the true and the nominal rates for the various periods are given in a foot note to each table.

(1.) Table I. gives the instalment payable at the end of each month, quarter, half-year, or year, to repay a loan of \$1,000 in any period from 1 to 20 years.

These instalments are given exact to the nearest cent above the true value when the difference exceeds one-tenth of a cent.

(2.) Table II. gives the present value of an instalment of \$1 payable at the end of each month, quarter, half-year, or year, during any number of years from 1 to 20.

By aid of this table, the present value of an instalment for an exact number of years is at once found by multiplying the instalment by the corresponding number in the table.

EXAMPLE.—Interest at 10 per cent. per annum, payable half-yearly. An instalment of \$20 is payable at the end of each month for 7 years: required, the present value.

No. corresponding in Table II.	60.6174
Instalment.....	20
Present value	<u>\$1212.35</u>

(3.) Table III. gives the present value of \$1 due at the end of any number of months from 1 to 12.

By aid of this Table and Table II., the present value of an instalment for a broken period of a number of years and some months can be found—as follows:

From Table II. take the present value of an instalment of \$1 for the named number of years, and discount it for the number of months by multiplying it by the factor corresponding to that number of months in Table III.; then add to it the present values taken from Table III. of the several instalments of \$1, payable during the broken period of months, and multiply the final sum by the given instalment.

EXAMPLE 1.—Interest at 10 per cent. per annum, payable half-yearly A yearly instalment of \$500 has 3 years and 9 months to run: required, its present value.

From Table II., present value of \$1 for 3 years 2.47595

Multiply this by the factor for 9 months from Table III. 0.9294284

giving 2.301218

Add the present value of an instalment \$1 due at the end

of 9 months, from Table III. 0.929428

3.230646

Multiply the instalment \$500 by this number, giving

Present value \$1615.32

EXAMPLE 2.—Same rate.

A half-yearly instalment of \$250 has 6 years and 11 months to run: required, its present value.

From Table II., present value of half-yearly instalment

\$1 for 6 years..... 8.8632

Multiply this by the factor for 11 months from Table III. 0.9144350

giving 8.10482

Brought forward	8.10482
Add the present value of an instalment \$1 due at the end of 5 months, from Table III.	0.96016
And the present value of an instalment \$1 due at the end of 11 months	0.91443
	9.97941

Multiply the instalment \$250 by this number, giving

Present value.... \$2494.85

EXAMPLE 3.—Required the present value of a quarterly instalment of \$100, unexpired period being 7 years and 5 months, interest at 11 per cent. per annum, payable half-yearly.

From Table II., present value of instalment of \$1 for

7 years, is 19.4395

Multiply this by the factor for 5 mos., from Table III. 0.9568633

giving 18.5912

Add the present value of \$1 due at end of 2 months.. 0.9823

And the present value of \$1 due at end of 5 months.. 0.9564

giving 20.5299

Multiply the instalment \$100 by this number, giving..

Present value.... \$2052.99

In the same way by aid of these two Tables, the present value of a monthly instalment for a broken period may be found, but this will be more conveniently effected by Table IV.

(4.) Table IV. gives the present values of a monthly instalment for any number of months not exceeding a year.

By aid of this and the previous tables, the present value of a monthly instalment for a broken period of a number of years and some months can be found, as follows:

From Table II. take the present value of an instalment of \$1 for the named number of years, and discount it for the number of months by multiplying it by the factor corresponding to that number of months in Table III; then add the present value of an instalment of \$1 from Table IV. for the number of months, and multiply the final sum by the given instalment.

EXAMPLE.—Interest 10 per cent. per annum, payable half-yearly.

An instalment of \$12.50 per month has 4 years and 8 months to run: required, its present value.

Present value of monthly instalment \$1 for 4 years, from
Table II. 39.5795

Multiply this by the factor for 8 months, from Table III.
0.9370174, giving 37.0866

Add present value of monthly instalment of \$1 for 8
months, from Table IV..... 7.7189

44.8005

Multiply the instalment \$12.50 by this number, giving
Present value.... \$560.61

(5.) Table V. gives the amount with interest of \$1 after any
number of months from 1 to 12.

By aid of this and Table II., the present value of an instalment
for a broken period of a number of years and some months can
also be found, as follows:

From Table II. take the present value of an instalment of \$1 for
the number of years next greater than the broken period, and find
the amount of it for the difference between the given number of
months and one year, by multiplying it by the factor correspond-
ing to that difference in Table V. Then subtract from it the
amounts (with interest, of the several instalments of \$1 (if any)
paid during that difference as given in Table V., and multiply the
result by the given instalment.

EXAMPLE 1.—Interest 10 per cent. per annum, payable half-
yearly. A yearly instalment of \$500 has 3 years and 9 months to
run: required, its present value.

From Table II., present value of yearly instalment of
\$1 for 4 years 3.15279

Multiply this by the factor for 3 months, from Table V.,
1.024695, giving..... 3.23065

No instalment has been paid during the 3 months.

Therefore, multiply the instalment \$500 by the above,
giving Present value \$1615.32

EXAMPLE 2.—A half-yearly instalment of \$250 has 6 years and 11 months to run: required, its present value. (Same rate.)

From Table II., present value of half-yearly instalment
of \$1 for 7 years, is 9.8986

Multiply this by the factor for 1 month, from Table V.,
1.008165, giving 9.9794

No instalment has been paid during the 1 month.

Therefore, multiply the instalment \$250 by the above,
giving Present value.... \$2494.85

EXAMPLE 3.—Required the present value of a quarterly instalment of \$100, unexpired period being 7 years and 5 months. (Same rate.)

From Table II., present value of quarterly instalment of
\$1 for 8 years, is 21.2082

Multiply this by the factor for 7 months, from Table V. 1.064456

giving 22.5751

Subtract the amount of the instalment of \$1
paid 4 months before 1.0363

And the amount of the instalment of \$1
paid 1 month before 1.0089 2.0452

giving 2.05299

Multiply the instalment \$100 by this number, giving
Present value.... \$2052.99

In the same way by the aid of these two Tables the present value of a monthly instalment for a broken period can be found; but this can be more conveniently effected by Table VI.

(6). Table VI. gives the amount at the end of any number of months, not exceeding a year, of a monthly instalment of \$1 paid at the end of each month during that period.

By aid of this, and Tables II. and V., the present value of a monthly instalment for a broken period of a number of years and some months can be found as follows: From Table II. take the present value of a monthly instalment of \$1 for the number of years next greater than the broken period, and find the amount of

it for the difference between the given number of months and one year by multiplying it by the factor corresponding to that difference in Table V.; then subtract from it the amount of an instalment of \$1 for that difference given in Table VI., and multiply the result by the given instalment.

EXAMPLE.—Interest 10 per cent. per annum, payable half-yearly.

An instalment of \$12.50 per month has four years and eight months to run: required, its present value.

Present value of monthly instalment of \$1 for 5 years,

from Table II. 47.2865

Multiply this by the factor for 4 months, from Table V.

1.033062, giving 48.8498

Subtract from this the amount corresponding to 4 months

in Table VI. 4.0498

giving 44.8005

Multiply the instalment \$12.50 by this number, giving.. \$560.01

(7). Table VIII. gives the present value of \$1 due at the end of any number of years, from 1 to 20.

With the aid of Table III. this also gives the present value of a sum due after a broken period of a number of years and some months, as follows;

Multiply the sum by the factor corresponding to the number of years in Table VIII., and then multiply this product by the factor corresponding to the number of months in Table III.

EXAMPLE. — Required the present value of \$4,000, due 17 years and 8 months hence, interest 10 per cent. payable half-yearly.

Multiply \$4,000 by the factor for 17 years, from Table

VIII., 0.1903548, giving 761.4192

Multiply this by the factor for 8 months from Table III. 0.9370174

Present value.. \$713.46

(8) By the aid of this and previous Tables, the value of an ordinary mortgage, where the principal is paid at the end of the period, can be calculated for any of the given rates of interest.

Consider the interest payable on the mortgage as an instalment and calculate its present value by the preceding rules; then add

to it the present value of the principal payable at the end of the period, which will be found as in the last example.

Ex. 1.—A mortgage of \$5,000, interest at 6 per cent. per annum, has 7 years and 10 months to run: find its present value, interest at 10 per cent. per annum, payable half-yearly.

The interest, \$300, may be considered a yearly instalment.

The present value of yearly instalment of \$1 for 7 years,

from Table II, is 4.82860

Multiply this by the factor for 10 months from Table III. 0.9219014

giving 4.451491

Add the present value of the instalment of \$1 due at

the end of 10 months 0.921901

giving 5.373392

Multiply the instalment \$300 by this number, giving

Present value.... \$1,612 02

Again: multiply the principal, \$5,000, by the factor for

7 years from Table VIII, 0.5050679, giving..... 2525.34

Multiply this by the factor for 10 mos. from Table III. 0.9219014

giving \$2,328 11

Adding these results, the present value required is.... \$3,940 13

Ex. 2.—A mortgage of \$4,000, interest at 5 per cent. per annum, payable half-yearly, has 17 years and 8 months to run: find its present value, interest 10 per cent. per annum, payable half-yearly.

The interest, \$100, will be considered a half-yearly instalment.

The present value of a half-yearly instalment of \$1 for

17 years, from Table II, is 16.1929

Multiply this by the factor for 8 months, from Table III. 0.9370174

giving 15.17303

Add the present value of instalment of \$1 due at end

of 2 months, from Table III. 0.98387

And the present value of the same, due at end of 8 mos. 0.93702

giving 17.09393

Multiply the interest, \$100, by this number, giving...	\$1,709 39
The present value of the principal, \$4,000, due 17 years and 8 months hence, is found, in the last example but one, to be.....	\$713 46
Adding these two results, we find the present value required to be	\$2,422 85

(9). Sometimes the borrower may wish to pay off part of his debt by a lump sum at some time during the running of his payments. In this case, having calculated the present value of his future payments, deduct from it the sum he pays down, and consider the remainder as a new loan. If the borrower wishes to continue his payments at a reduced rate for the same period, the new rate of instalment can be obtained at once from Table I, when the period is an exact number of years; but if he wishes to continue to pay the same amount of instalment, reducing the period during which the payments are to continue, obtain from Tables II. and III. the period which will give for that instalment a present value next less than the above-mentioned remainder; take the difference between this present value and such remainder; this difference improved for the period at the given rate of interest will be an additional sum, which he must pay along with the final instalment to clear the account. To obtain this difference so improved, if for a broken period of a number of years and some months, *divide* the difference by the factor corresponding to the number of years in Table VIII, and then divide this quotient by the factor corresponding to the number of months in Table III.

EXAMPLE 1. — Interest at 10 per cent. per annum, payable half-yearly. A monthly instalment of \$20 as a loan has 5 years to run, and the borrower wishes to pay down \$500 along with his instalment: to find the reduced instalment for the remaining period.

The present value of the instalments from Table II.	\$945 73
Deduct.....	500 00
Remainder	445 73

Multiply this by the number for 5 years from Table I, 21.15, and dividing by 1000, we obtain \$9 43 for the reduced instalment,

EXAMPLE 2.—Interest as above.

A monthly instalment of \$10 has 7 years and 3 months to run, and the borrower wishes to pay down along with his instalment \$200: to find the reduction in the period.

Present value of instalments, found as in (3)	\$621 08
Deduct.....	200 00
Remainder	421 08

The present value of a monthly instalment, \$10, is now to be found, which is next below \$421 08 in the Tables.

From Table II. it is seen that the present value of a monthly instalment of \$10 for 4 years is \$395 80, and on trial with Table III. it is found that the present value of an instalment of \$10 for a period of 4 years and 3 months is \$415 78, and is the next below the given remainder \$421 08. Hence the period required is 4 years and 3 months, and the remaining difference \$5 30, amounting in this period to \$8 03, leaves this additional sum to be paid along with the final instalment.

(10). Given the loan, the instalment, and the number of years for repayment: it is required to determine the rate of interest.

When the instalment for \$1,000 lies between two values in the Tables, an approximate value of the rate of interest can at once be obtained by interpolation, and this will in general be near enough for practical purposes; or from this approximate value, closer and closer values may be obtained by the method of "trial and error," or "double position."

When the instalment for \$1,000 is beyond the Tables, the method of proceeding will be found in the Appendix.

INTEREST 8 PER CENT. PER ANNUM, PAYABLE HALF-YEARLY.

TABLE I.

Instalments to repay a Loan of \$1000 in the named number of years, payable

Years.	Yearly.	Half-yearly.	Quarterly.	Monthly.	Years.
1	1081.60	530.20	262.50	86.93	1
2	562.00	275.49	136.40	45.17	2
3	389.16	190.77	94.45	31.28	3
4	303.00	148.53	73.54	24.36	4
5	251.52	123.29	61.04	20.22	5
6	217.37	106.56	52.76	17.47	6
7	193.13	94.67	46.87	15.53	7
8	175.08	85.82	42.49	14.07	8
9	161.15	79.00	39.11	12.96	9
10	150.11	73.59	36.43	12.07	10
11	141.17	69.20	34.26	11.35	11
12	133.80	65.59	32.48	10.76	12
13	127.64	62.57	30.98	10.26	13
14	122.43	60.02	29.72	9.84	14
15	117.93	57.83	28.64	9.49	15
16	114.14	55.95	27.70	9.18	16
17	110.81	54.32	26.90	8.91	17
18	107.89	52.89	26.19	8.68	18
19	105.33	51.64	25.51	8.47	19
20	103.07	50.53	25.02	8.29	20

NOMINAL RATES OF INTEREST : 8 per cent. per annum payable half-yearly ; 7.9216 per cent. per annum payable quarterly ; 7.8703 per cent. per annum payable monthly.

ACTUAL RATES : 8.16 per cent. per annum ; 4 per cent. per half-year ; 1.9804 per cent. per quarter ; 0.6559 per cent. per month.

INTEREST 8 PER CENT. PER ANNUM, PAYABLE HALF-YEARLY.

TABLE II.

Present Value of Instalment of \$1 payable

Years.	Yearly.	Half-Yearly.	Quarterly.	Monthly.	Years.
1	0.92456	1.8861	3.8095	11.5037	1
2	1.77936	3.6299	7.3317	22.1396	2
3	2.56968	5.2421	10.5881	31.9730	3
4	3.30037	6.7327	13.5988	41.0646	4
5	3.97593	8.1109	16.3824	49.4703	5
6	4.60053	9.3851	18.9560	57.2418	6
7	5.17800	10.5631	21.3354	64.4270	7
8	5.71191	11.6523	23.5353	71.0701	8
9	6.20554	12.6593	25.5693	77.2121	9
10	6.66192	13.5903	27.4498	82.8906	10
11	7.08388	14.4511	29.1884	88.1408	11
12	7.47400	15.2470	30.7959	92.9948	12
13	7.83469	15.9828	32.2821	97.4827	13
14	8.16817	16.6631	33.6561	101.6320	14
15	8.47649	17.2920	34.9265	105.4682	15
16	8.76154	17.8735	36.1011	109.0150	16
17	9.02510	18.4112	37.1870	112.2943	17
18	9.26876	18.9083	38.1910	115.3261	18
19	9.49405	19.3679	39.1193	118.1292	19
20	9.70234	19.7928	39.9775	120.7208	20

INTEREST 8 PER CENT. PER ANNUM, PAYABLE HALY-YEARLY.

TABLE III.

Present value of \$1 due after any number of months, from 1 to 12.

Months.	Present Value.
1	0.9934845
2	0.9870115
3	0.9805808
4	0.9741915
5	0.9678443
6	0.9615386
7	0.9552736
8	0.9490496
9	0.9428660
10	0.9367229
11	0.9306197
12	0.9245563

TABLE IV.

The present value of an Instalment of \$1, payable at the end of each month during the named number of months.

Months.	Present Value.
1	0.993485
2	1.980496
3	2.961077
4	3.935263
5	4.903113
6	5.864651
7	6.819925
8	7.768974
9	8.711840
10	9.648563
11	10.579183
12	11.503739

INTEREST 8 PER CENT. PER ANNUM, PAYABLE HALF-YEARLY.

TABLE V.

Amount of \$1 in any number of months, from 1 to 12.

Months.	Amounts.
1	1.006559
2	1.013160
3	1.019804
4	1.026492
5	1.033225
6	1.040000
7	1.046820
8	1.053685
9	1.060596
10	1.067552
11	1.074552
12	1.081600

TABLE VI.

Amount at the end of any number of months, from 1 to 12, of a Monthly Instalment of \$1, paid at the end of each month during that period.

Months.	Amounts.
1	1.000000
2	2.006559
3	3.019719
4	4.039523
5	5.066015
6	6.099240
7	7.139240
8	8.186060
9	9.239745
10	10.300341
11	11.367893
12	12.442445

INTEREST 8 PER CENT. PER ANNUM, PAYABLE HALF-YEARLY.

TABLE VII.

Amount at the end of any number of quarters, from 1 to 4, of a quarterly Instalment of \$1, paid at the end of each quarter during that period.

Qrs.	Amounts.
1	1.000000
2	2.019804
3	3.059804
4	4.120400

TABLE VIII.

Present Value of \$1 due at the end of any number of years, from 1 to 20.

Yrs.	Present Values.	Yrs.	Present Values.
1	0.924556	11	0.421955
2	0.854804	12	0.390121
3	0.790315	13	0.360689
4	0.730690	14	0.333477
5	0.675564	15	0.308319
6	0.624597	16	0.285058
7	0.577475	17	0.263552
8	0.533908	18	0.243669
9	0.493628	19	0.225285
10	0.456387	20	0.208289

INTEREST 9 PER CENT. PER ANNUM, PAYABLE HALF-YEARLY.

TABLE I.

Instalments to repay a Loan of \$1000 in the named number of years, payable

Years.	Yearly.	Half-yearly.	Quarterly.	Monthly.	Years.
1	1092.03	534.00	264.06	87.38	1
2	570.04	278.75	137.84	45.61	2
3	396.48	193.88	95.88	31.73	3
4	310.05	151.61	74.97	24.81	4
5	258.45	126.38	62.50	20.68	5
6	224.27	109.67	54.23	17.95	6
7	200.05	97.82	48.38	16.01	7
8	182.04	89.02	44.02	14.57	8
9	168.18	82.24	40.67	13.46	9
10	157.22	76.88	38.02	12.58	10
11	148.36	72.55	35.88	11.87	11
12	141.08	68.99	34.12	11.29	12
13	135.02	66.03	32.65	10.81	13
14	129.90	63.52	31.41	10.40	14
15	125.55	61.40	30.36	10.05	15
16	121.81	59.57	29.46	9.75	16
17	118.58	57.99	28.68	9.49	17
18	115.76	56.61	28.00	9.27	18
19	113.30	55.41	27.40	9.07	19
20	111.14	54.35	26.88	8.90	20

NOMINAL RATE OF INTEREST: 9 per cent. per annum payable half-yearly; 8.9008 per cent. per annum payable quarterly; 8.8356 per cent. per annum payable monthly.

ACTUAL RATE: 9.2025 per cent. per annum; $4\frac{1}{2}$ per cent. per half-year; 2.2252 per cent. per quarter; 0.7363 per cent. per month.

INTEREST 9 PER CENT. PER ANNUM, PAYABLE HALF-YEARLY.

TABLE II.

Present Value of Instalment of \$1 payable

Years	Yearly.	Half-Yearly.	Quarterly.	Monthly.	Years.
1	0.91573	1.8727	3.7870	11.4449	1
2	1.75429	3.5875	7.2549	21.9253	2
3	2.52219	5.1579	10.4305	31.5225	3
4	3.22537	6.5959	13.3385	40.3110	4
5	3.86930	7.9127	16.0015	48.3589	5
6	4.45896	9.1186	18.4401	55.7285	6
7	4.99894	10.2228	20.6731	62.4772	7
8	5.49341	11.2340	22.7180	68.6571	8
9	5.94621	12.160	24.5906	74.3162	9
10	6.36085	13.0079	26.3053	79.4985	10
11	6.74055	13.7844	27.8756	84.2440	11
12	7.08825	14.4955	29.3135	88.5896	12
13	7.40666	15.1466	30.6303	92.5690	13
14	7.69823	15.7429	31.8361	96.2131	14
15	7.96523	16.2889	32.9402	99.5501	15
16	8.20973	16.7889	33.9514	102.6059	16
17	8.43362	17.2468	34.8773	105.4042	17
18	8.63865	17.6660	35.7252	107.9666	18
19	8.82640	18.0500	36.5016	110.3131	19
20	8.99833	18.4016	37.2126	112.4619	20

INTEREST 9 PER CENT. PER ANNUM, PAYABLE HALF-YEARLY.

TABLE III.

*Present value of \$1 due after
any number of months, from
1 to 12.*

Months.	Present Values.
1	0.9926908
2	0.9854349
3	0.9782321
4	0.9710818
5	0.9639839
6	0.9569378
7	0.9499433
8	0.9429998
9	0.9361072
10	0.9292650
11	0.9224722
12	0.9157301

TABLE IV.

*The present value of an Instal-
ment of \$1, payable at the end
of each month during the named
number of months.*

Months.	Present Values.
1	0.992691
2	1.978126
3	2.956358
4	3.927440
5	4.891423
6	5.848361
7	6.798305
8	7.741304
9	8.677412
10	9.606677
11	10.529149
12	11.444879

INTEREST 9 PER CENT. PER ANNUM, PAYABLE HALF-YEARLY.

TABLE V.

Amount of \$1 in any number of months from 1 to 12.

Months.	Amounts.
1	1.007363
2	1.014780
3	1.022252
4	1.029779
5	1.037362
6	1.045000
7	1.052695
8	1.060446
9	1.068254
10	1.076119
11	1.084044
12	1.092025

TABLE VI.

Amount at the end of any number of months, from 1 to 12, of a monthly Instalment of \$1, paid at the end of each month during that period.

Months.	Amounts.
1	1.000000
2	2.007363
3	3.022143
4	4.044395
5	5.074174
6	6.111536
7	7.156536
8	8.209231
9	9.269677
10	10.337931
11	11.414050
12	12.498094

INTEREST 9 PER CENT. PER ANNUM, PAYABLE HALF-YEARLY.

TABLE VII.

Amount at the end of any number of quarters, from 1 to 4, of a quarterly Instalment of \$1, paid at the end of each quarter during that period.

Qrs.	Amounts.
1	1.000000
2	2.022252
3	3.067252
4	4.135506

TABLE VIII.

Present value of \$1 due at the end of any number of years from 1 to 20.

Yrs.	Present Values.	Yrs.	Present Values.
1	0.9157300	11	0.3797009
2	0.8385613	12	0.3477035
3	0.7678957	13	0.3184025
4	0.7031851	14	0.2915707
5	0.6439277	15	0.2670000
6	0.5896639	16	0.2444999
7	0.5399729	17	0.2238959
8	0.4944693	18	0.2050282
9	0.4528004	19	0.1877504
10	0.4146429	20	0.1719287

INTEREST 10 PER CENT. PER ANNUM, PAYABLE HALF-YEARLY.

TABLE I.

Instalments to repay a Loan of \$1000, in the named number of years, payable

Years.	Yearly.	Half-yearly.	Quarterly.	Monthly.	Years.
1	1102.50	537.81	265.63	87.83	1
2	578.13	282.02	139.29	46.06	2
3	403.89	197.02	97.31	32.18	3
4	317.18	154.73	76.42	25.27	4
5	265.49	129.51	63.97	21.15	5
6	231.30	112.83	55.73	18.43	6
7	207.10	101.03	49.90	16.50	7
8	189.16	92.27	45.58	15.07	8
9	175.37	85.55	42.26	13.97	9
10	164.50	80.25	39.64	13.11	10
11	155.74	75.98	37.53	12.41	11
12	148.57	72.48	35.80	11.84	12
13	142.61	69.57	34.36	11.36	13
14	137.61	67.13	33.16	10.96	14
15	133.36	65.06	32.13	10.63	15
16	129.73	63.28	31.26	10.34	16
17	126.60	61.76	30.51	10.09	17
18	123.89	60.44	29.85	9.87	18
19	121.54	59.29	29.28	9.68	19
20	119.47	58.28	28.79	9.52	20

NOMINAL RATES OF INTEREST : 10 per cent. per annum, payable half-yearly ; 9.86364 per cent. per annum, payable quarterly ; 9.71788 per cent. per annum, payable monthly.

ACTUAL RATES : 10.25 per cent. per annum ; 5 per cent. per half-year 2.46591 per cent. per quarter ; 0.81649 per cent. per month.

INTEREST 10 PER CENT. PER ANNUM, PAYABLE HALF-YEARLY.

TABLE II.

Present Value of Instalment of \$1 payable

Years.	Yearly.	Half-Yearly.	Quarterly.	Monthly.	Years.
1	0.90703	1.8594	3.7647	11.3367	1
2	1.72973	3.5460	7.1795	21.7147	2
3	2.47595	5.0757	10.2767	31.0826	3
4	3.15279	6.4632	13.0860	39.5795	4
5	3.76670	7.7217	15.6342	47.2865	5
6	4.32354	8.8632	17.9454	54.2769	6
7	4.82860	9.8986	20.0417	60.6174	7
8	5.28672	10.8378	21.9432	66.3685	8
9	5.70224	11.6896	23.6679	71.5848	9
10	6.07913	12.4622	25.2322	76.3163	10
11	6.42098	13.1630	26.6511	80.6078	11
12	6.73104	13.7986	27.9381	84.5003	12
13	7.01229	14.3752	29.1054	88.0310	13
14	7.26738	14.8981	30.1642	91.2334	14
15	7.49876	15.3725	31.1245	94.1380	15
16	7.70862	15.8027	31.9956	96.7726	16
17	7.89898	16.1929	32.7857	99.1623	17
18	8.07163	16.5469	33.5023	101.3298	18
19	8.22824	16.8679	34.1523	103.2958	19
20	8.37029	17.1591	34.7419	105.0790	20

IN

INTEREST 10 PER CENT. PER ANNUM, PAYABLE HALF-YEARLY.

TABLE III.

Present value of \$1 due after any number of months from 1 to 12.

Months.	Present Values.
1	0.9919018
2	0.9838680
3	0.9759000
4	0.9679964
5	0.9601569
6	0.9523809
7	0.9446679
8	0.9370174
9	0.9294284
10	0.9219014
11	0.9144350
12	0.9070294

TABLE IV.

The present value of an Instalment of \$1, payable at the end of each month during the named number of months.

Months.	Present Values.
1	0.991901
2	1.975769
3	2.951669
4	3.919666
5	4.879823
6	5.832203
7	6.776871
8	7.713889
9	8.643317
10	9.565219
11	10.479654
12	11.386683

INTEREST 10 PER CENT. PER ANNUM, PAYABLE HALF-YEARLY.

IV.

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s.

Values.

91901

75769

1669

9666

9823

2203

871

889

317

219

554

83

TABLE V.

Amount of \$1 in any number of
months from 1 to 12.

Months.	Amounts.
1	1.008165
2	1 016397
3	1.024695
4	1.033062
5	1.041496
6	1.050006
7	1.058573
8	1.067216
9	1.075930
10	1.084724
11	1.093571
12	1.102500

TABLE VI.

Amount at the end of any number
of months from 1 to 12, of a
monthly instalment of \$1 paid
at the end of each month during
that period.

Months.	Amounts.
1	1.000000
2	2.008165
3	3.024562
4	4.049257
5	5.082319
6	6.123815
7	7.173815
8	8.232388
9	9 299604
10	10.375534
11	11.460248
12	12.553819

INTEREST 10 PER CENT. PER ANNUM, PAYABLE HALF-YEARLY.

TABLE VII.

Amount at the end of any number of quarters from 1 to 4 of a quarterly instalment of \$1, paid at the end of each quarter during that period.

Qrs.	Amounts.
1	1.000000
2	2.024695
3	3.074695
4	4.150624

TABLE VIII.

Present value of \$1 due at the end of any number of years, from 1 to 20.

Yrs.	Present Values.	Yrs.	Present Values.
1	0.9070295	11	0.3418499
2	0.8227025	12	0.3100679
3	0.7462154	13	0.2812407
4	0.6768394	14	0.2550936
5	0.6139132	15	0.2313774
6	0.5568374	16	0.2098662
7	0.5050679	17	0.1903548
8	0.4581115	18	0.1720574
9	0.4155206	19	0.1566054
10	0.3768895	20	0.1420457

INTEREST 11 PER CENT. PER ANNUM, PAYABLE HALF-YEARLY.

TABLE I.

Instalments to repay a Loan of \$1000 in the named number of years, payable

Years.	Yearly.	Half-yearly.	Quarterly.	Monthly.	Years.
1	1113.03	541.62	267.19	88.27	1
2	586.29	285.30	140.74	46.50	2
3	411.37	200.18	98.75	32.63	3
4	324.42	157.87	77.88	25.73	4
5	272.64	132.67	65.45	21.63	5
6	238.44	116.03	57.24	18.91	6
7	214.30	104.28	51.45	17.00	7
8	196.43	95.59	47.16	15.58	8
9	182.74	88.92	43.87	14.50	9
10	171.97	83.68	41.28	13.64	10
11	163.32	79.48	39.21	12.96	11
12	156.26	76.04	37.51	12.40	12
13	150.42	73.20	36.11	11.93	13
14	145.53	70.82	34.94	11.54	14
15	141.40	68.81	33.95	11.22	15
16	137.88	67.10	33.10	10.94	16
17	134.87	65.63	32.38	10.70	17
18	132.28	64.37	31.76	10.49	18
19	130.03	63.28	31.22	10.32	19
20	128.07	62.32	30.75	10.16	20

NOMINAL RATE OF INTEREST: 11 per cent. per annum payable half-yearly; 10.85276 per cent. per annum payable quarterly; 10.75608 per cent. per annum payable monthly.

ACTUAL RATE: 11.3025 per cent. per annum; $5\frac{1}{2}$ per cent. per half-year; 2.71319 per cent. per quarter; 0.89634 per cent. per month.

INTEREST 11 PER CENT. PER ANNUM, PAYABLE HALF-YEARLY.

TABLE II.

Present Value of Instalment of \$1 payable

Years	Yearly.	Half-Yearly.	Quarterly.	Monthly.	Years.
1	0.89845	1.8463	3.7427	11.3291	1
2	1.70567	3.5052	7.1054	21.5078	2
3	2.43092	4.9955	10.1266	30.6529	3
4	3.08252	6.3345	12.8410	38.8693	4
5	3.66795	7.5376	15.2798	46.2514	5
6	4.19393	8.6185	17.4709	52.8838	6
7	4.66650	9.5897	19.4395	58.8427	7
8	5.09108	10.4622	21.2082	64.1965	8
9	5.47254	11.2461	22.7973	69.0067	9
10	5.81527	11.9504	24.2250	73.3284	10
11	6.12320	12.5832	25.5077	77.2112	11
12	6.39986	13.1517	26.6602	80.6997	12
13	6.64841	13.6625	27.6957	83.8340	13
14	6.87174	14.1214	28.6260	86.6500	14
15	7.07238	14.5337	29.4618	89.1801	15
16	7.25265	14.9042	30.2128	91.4532	16
17	7.41461	15.2370	30.8875	93.4955	17
18	7.56013	15.5361	31.4937	95.3304	18
19	7.69087	15.8047	32.0383	96.9790	19
20	7.80833	16.0461	32.5276	98.4601	20

INTEREST 11 PER CENT. PER ANNUM, PAYABLE HALF-YEARLY.

TABLE III.

Present value of \$1 due after any number of months from 1 to 12.

Months.	Present Value.
1	0.9911162
2	0.9823114
3	0.9735848
4	0.9649356
5	0.9563633
6	0.9478673
7	0.9394467
8	0.9311009
9	0.9228292
10	0.9146310
11	0.9065056
12	0.8984524

TABLE IV.

The present value of an Instalment of \$1, payable at the end of each month during the named number of months.

Months.	Present Value.
1	0.991116
2	1.973428
3	2.947012
4	3.911948
5	4.868311
6	5.816179
7	6.755625
8	7.686726
9	8.609555
10	9.524186
11	10.430692
12	11.329144

 INTEREST 11 PER CENT. PER ANNUM, PAYABLE HALF-YEARLY.

TABLE V.

Amount of \$1 in any number of months from 1 to 12.

Months.	Amounts.
1	1.008963
2	1.018008
3	1.027132
4	1.036338
5	1.045628
6	1.055000
7	1.064456
8	1.073997
9	1.083624
10	1.093337
11	1.103137
12	1.113025

TABLE VI.

Amount at the end of any number of months, from 1 to 12, of a monthly Instalment of \$1, paid at the end of each month during that period.

Months.	Amounts.
1	1.000000
2	2.008963
3	3.026971
4	4.054103
5	5.090441
6	6.136069
7	7.191069
8	8.255525
9	9.329522
10	10.413146
11	11.506483
12	12.609620

INTEREST 11 PER CENT. PER ANNUM, PAYABLE HALF-YEARLY.

TABLE VII.

Amount at the end of any number of quarters, from 1 to 4, of a quarterly Instalment of \$1, paid at the end of each quarter during that period.

Q	s.	Amounts.
1		1.000000
2		2.027132
3		3.082132
4		4.165756

TABLE VIII.

Present value of \$1 due at the end of any number of years from 1 to 20.

Yrs.	Present Values.	Yrs.	Present Values.
1	0.8984524	11	0.3079256
2	0.8072162	12	0.2766566
3	0.7252458	13	0.2485627
4	0.6515982	14	0.2233218
5	0.5854305	15	0.2006440
6	0.5259816	16	0.1802691
7	0.4725693	17	0.1619632
8	0.4245811	18	0.1455162
9	0.3814659	19	0.1307394
10	0.3427290	20	0.1174631

INTEREST 12 PER CENT. PER ANNUM, PAYABLE HALF-YEARLY.

TABLE I.

Instalments to repay a Loan of \$1000, in the named number of years, payable

Years.	Yearly.	Half-yearly.	Quarterly.	Monthly.	Years.
1	1123.60	545.44	268.75	88.72	1
2	594.50	288.60	142.20	46.94	2
3	418.93	203.37	100.21	33.08	3
4	331.74	161.04	79.35	26.20	4
5	279.89	135.87	66.95	22.10	5
6	245.72	119.28	58.77	19.40	6
7	221.63	107.59	53.01	17.50	7
8	203.85	98.96	48.76	16.10	8
9	190.26	92.36	45.51	15.03	9
10	179.61	87.19	42.96	14.19	10
11	171.03	83.05	40.92	13.51	11
12	164.14	79.68	39.26	12.96	12
13	158.43	76.91	37.90	12.51	13
14	153.66	74.60	36.76	12.14	14
15	149.66	72.65	35.80	11.82	15
16	146.27	71.01	34.99	11.55	16
17	143.38	69.60	34.30	11.32	17
18	140.90	68.40	33.70	11.13	18
19	138.76	67.36	33.19	10.96	19
20	136.91	66.47	32.75	10.81	20

NOMINAL RATES OF INTEREST: 12 per cent. per annum, payable half-yearly; 11.8252 per cent. per annum, payable quarterly; 11.71056 per cent. per annum, payable monthly.

ACTUAL RATES: 12.36 per cent. per annum; 6 per cent. per half-year; 2.9563 per cent. per quarter; 0.97588 per cent. per month.

INTEREST 12 PER CENT. PER ANNUM, PAYABLE HALF-YEARLY.

TABLE II.

Present Value of Instalment of \$1 payable

Years.	Yearly.	Half-Yearly.	Quarterly.	Monthly.	Years.
1	0.89000	1.8334	3.7210	11.2722	1
	1.68209	3.4651	7.0326	21.3045	2
3	2.38705	4.9173	9.9800	30.2332	3
4	3.01446	6.2098	12.6032	38.1797	4
5	3.57286	7.3601	14.9378	45.2520	5
6	4.06983	8.3838	17.0155	51.5464	6
7	4.51213	9.2950	18.8648	57.1484	7
8	4.90577	10.1059	20.5106	62.1341	8
9	5.25612	10.8276	21.9753	66.5714	9
10	5.56792	11.4699	23.2789	70.5205	10
11	5.84543	12.0416	24.4392	74.0353	11
12	6.09241	12.5504	25.4717	77.1634	12
13	6.31222	13.0032	26.3908	79.9474	13
14	6.50785	13.4062	27.2087	82.4251	14
15	6.68196	13.7648	27.9366	84.6303	15
16	6.83691	14.0840	28.5845	86.5929	16
17	6.97483	14.3681	29.1611	88.3297	17
18	7.09757	14.6210	29.6742	89.8942	18
19	7.20681	14.8460	30.1309	91.2778	19
20	7.30403	15.0463	30.5374	92.5092	20

INTEREST 12 PER CENT. PER ANNUM, PAYABLE HALF-YEARLY.

TABLE III.

Present value of \$1 due after any number of months, from 1 to 12.

Months.	Present Values.
1	0.9903356
2	0.9807645
3	0.9712858
4	0.9618989
5	0.9526026
6	0.9433960
7	0.9342786
8	0.9252495
9	0.9163074
10	0.9074517
11	0.8986816
12	0.8899965

TABLE IV.

The present value of an Instalment of \$1, payable at the end of each month during the named number of months.

Months.	Present Values.
1	0.990336
2	1.971100
3	2.942386
4	3.904285
5	4.856887
6	5.800283
7	6.734562
8	7.659812
9	8.576119
10	9.483571
11	10.382252
12	11.272249

INTEREST 12 PER CENT. PER ANNUM, PAYABLE HALF-YEARLY.

TABLE V.

Amount of \$1 in any number of months, from 1 to 12.

Months.	Amounts.
1	1.009759
2	1.019612
3	1.029562
4	1.039610
5	1.049756
6	1.060000
7	1.070344
8	1.080789
9	1.091338
10	1.101987
11	1.112741
12	1.123600

TABLE VI.

Amount at the end of any number of months, from 1 to 12, of a Monthly Instalment of \$1, paid at the end of each month during that period.

Months.	Amounts.
1	1.000000
2	2.009759
3	3.029371
4	4.058933
5	5.098543
6	6.148299
7	7.208299
8	8.278643
9	9.359432
10	10.450770
11	11.552757
12	12.665498

INTEREST 12 PER CENT. PER ANNUM, PAYABLE HALF-YEARLY.

TABLE VII.

Amount at the end of any number of quarters, from 1 to 4, of a quarterly Instalment of \$1, paid at the end of each quarter during that period.

Qrs.	Amounts.
1	1.000000
2	2.029562
3	3.089562
4	4.180900

TABLE VIII.

Present Value of \$1 due at the end of any number of years, from 1 to 20.

Yrs.	Present Values.	Yrs.	Present Values.
1	0.889996	11	0.277505
2	0.792094	12	0.246979
3	0.704961	13	0.219810
4	0.627412	14	0.195630
5	0.558395	15	0.174110
6	0.496969	16	0.154957
7	0.442301	17	0.137912
8	0.393646	18	0.122741
9	0.350344	19	0.109239
10	0.311805	20	0.097222

INTEREST 13 PER CENT. PER ANNUM, PAYABLE HALF-YEARLY.

TABLE I.

Instalments to repay a Loan of \$1000 in the named number of years, payable

Years.	Yearly.	Half-yearly.	Quarterly.	Monthly.	Years.
1	1134.23	549.27	270.31	89.16	1
2	602.78	291.91	143.66	47.39	2
3	426.57	206.57	101.66	33.54	3
4	339.15	164.24	80.83	26.66	4
5	287.26	139.11	68.46	22.58	5
6	253.11	122.57	60.32	19.90	6
7	229.10	110.94	54.60	18.01	7
8	211.41	102.38	50.39	16.62	8
9	197.94	95.86	47.18	15.56	9
10	187.42	90.76	44.67	14.74	10
11	179.02	86.70	42.67	14.08	11
12	172.22	83.40	41.05	13.54	12
13	166.64	80.70	39.72	13.10	13
14	162.01	78.46	38.61	12.74	14
15	158.14	76.58	37.69	12.43	15
16	154.87	75.00	36.91	12.18	16
17	152.10	73.66	36.25	11.96	17
18	149.74	72.52	35.69	11.77	18
19	147.72	71.54	35.21	11.62	19
20	145.99	70.70	34.79	11.48	20

NOMINAL RATES OF INTEREST : 13 per cent. per annum payable half-yearly ; 12.7956 per cent. per annum payable quarterly ; 12.6612 per cent. per annum payable monthly.

ACTUAL RATES : 13.4225 per cent. per annum ; $6\frac{1}{2}$ per cent. per half-year ; 3.1939 per cent. per quarter ; 1.0551 per cent. per month.

INTEREST 13 PER CENT. PER ANNUM, PAYABLE HALF-YEARLY.

TABLE II.

Present value of Instalment of \$1 payable

Years.	Yearly.	Half-yearly.	Quarterly.	Monthly.	Years.
1	0.88166	1.8206	3.6995	11.2160	1
2	1.65898	3.4258	6.9612	21.1047	2
3	2.34432	4.8410	9.8369	29.8231	3
4	2.94855	6.0888	12.3723	37.5098	4
5	3.48127	7.1888	14.6076	44.2869	5
6	3.95096	8.1587	16.5784	50.2619	6
7	4.36506	9.0138	18.3160	55.5299	7
8	4.73015	9.7678	19.8480	60.1744	8
9	5.05204	10.4325	21.1987	64.2693	9
10	5.33584	11.0185	22.3895	67.8796	10
11	5.58605	11.5352	23.4394	71.0627	11
12	5.80665	11.9907	24.3650	73.8691	12
13	6.00115	12.3924	25.1812	76.3433	13
14	6.17263	12.7465	25.9007	78.5248	14
15	6.32381	13.0587	26.5351	80.4481	15
16	6.45711	13.3339	27.0944	82.1438	16
17	6.57463	13.5766	27.5875	83.6388	17
18	6.67824	13.7906	28.0223	84.9569	18
19	6.76959	13.9792	28.4056	86.1191	19
20	6.85013	14.1455	28.7436	87.1437	20

INTEREST 13 PER CENT. PER ANNUM, PAYABLE HALF-YEARLY.

TABLE III.

Present value of \$1 due after any number of months, from 1 to 12.

Months.	Present Value.
1	0.9895592
2	0.9792270
3	0.9690032
4	0.9588859
5	0.9488743
6	0.9389672
7	0.9291635
8	0.9194623
9	0.9098622
10	0.9003621
11	0.8909618
12	0.8816594

TABLE IV.

The present value of an Instalment of \$1, payable at the end or each month during the named number of months.

Months.	Present Values.
1	0.989559
2	1.968786
3	2.937789
4	3.896675
5	4.845550
6	5.784517
7	6.713680
8	7.633143
9	8.543005
10	9.443367
11	10.334329
12	11.215988

INTEREST 13 PER CENT. PER ANNUM, PAYABLE HALF-YEARLY.

TABLE V.

Amount of \$1 in any number of months from 1 to 12.

Months.	Amounts.
1	1.010551
2	1.021213
3	1.031989
4	1.042876
5	1.053881
6	1.065000
7	1.076237
8	1.087593
9	1.099068
10	1.110665
11	1.122383
12	1.134225

TABLE VI.

Amount at the end of any number of months, from 1 to 12, of a Monthly Instalment of \$1, paid at the end of each month during that period.

Months.	Amounts.
1	1.000000
2	2.010551
3	3.031764
4	4.063753
5	5.106629
6	6.160510
7	7.225510
8	8.301747
9	9.389340
10	10.488408
11	11.599073
12	12.721456

INTEREST 13 PER CENT. PER ANNUM, PAYABLE HALF-YEARLY.

TABLE VII.

Amount at the end of any number of quarters, from 1 to 4, of a quarterly Instalment of \$1, paid at the end of each quarter during that period.

Qrs.	Amounts.
1	1.000000
2	2.081989
3	3.096989
4	4.196057

TABLE VIII.

Present value of \$1 due at the end of any number of years from 1 to 20.

Yrs.	Present Values.	Yrs.	Present Values.
1	0.881659	11	0.250212
2	0.777323	12	0.220602
3	0.685334	13	0.194496
4	0.604231	14	0.171479
5	0.532726	15	0.151186
6	0.469683	16	0.133295
7	0.414100	17	0.117520
8	0.365095	18	0.103611
9	0.321890	19	0.091351
10	0.283797	20	0.080541

APPENDIX.

NOTE I.—If i is the yearly rate of interest per unit, and $R = 1 + i$, the amount (M) of a sum P in n years is given by

$$M = PR^n$$

and the present value (P) of a sum (M) due n years hence is given by

$$P = MR^{-n}.$$

In these formulas n may be integral or fractional; thus, the amount of 1 in the p th part of a year is $R^{\frac{1}{p}}$, and the rate of interest per the p th part of the year is $R^{\frac{1}{p}} - 1$.

Hence the rate per half-year is $R^{\frac{1}{2}} - 1$.

“ “ quarter is $R^{\frac{1}{4}} - 1$.

“ “ month is $R^{\frac{1}{12}} - 1$.

But if i is the *nominal* yearly rate of interest per unit, payable p times a year, meaning thereby that $\frac{i}{p}$ is the interest payable at the end of each p th part of a year, then the amount of 1 in a year is $\left(1 + \frac{i}{p}\right)^p$, and the true yearly rate of interest is $\left(1 + \frac{i}{p}\right)^p - 1$.

NOTE II.—If A is an instalment payable at the end of each year for n years from the present time, and P its present value, then

$$P = A \cdot \frac{1 - R^{-n}}{R - 1}$$

in which formula n is necessarily a whole number.

If A is payable at the end of each of p equal intervals in a year, and the payments continue for n years, then

$$P = A \cdot \frac{1 - R^{-n}}{R^{\frac{1}{p}} - 1}$$

where n may be fractional, but such that the whole time contains an exact number of the intervals.

Thus if the instalment A is payable

$$\text{half-yearly, then } P = A \cdot \frac{1 - R^{-n}}{R^{\frac{1}{2}} - 1}$$

$$\text{quarterly, " } P = A \cdot \frac{1 - R^{-n}}{R^{\frac{1}{4}} - 1}$$

$$\text{monthly, " } P = A \cdot \frac{1 - R^{-n}}{R^{\frac{1}{12}} - 1}$$

These formulas give the relation between the loan and the instalment, and also give the present value of an instalment having any exact number of periods to run.

If the time to run is not an exact number of periods, the present value will be found by taking the present value for the whole number of periods less than the given period, discounting this for the broken interval, and adding the present value of the instalment payable at the end of the broken period. Thus, if a yearly instalment A has n years and m months to run, its present value is given by

$$P = AR^{-\frac{m}{12}} \cdot \frac{1 - R^{-n}}{R - 1} + AR^{-\frac{m}{12}}$$

NOTE III.—If v is the present value of 1 due at the end of any interval, and A is an instalment payable at the end of each of n such intervals, then

$$P = A (v + v^2 + v^3 + \dots \text{ to } n \text{ terms.})$$

If the interval is a year, then $v = R^{-1}$; if a half year, $v = R^{-\frac{1}{2}}$;

if a quarter, $v = R^{-\frac{1}{4}}$; if a month, $v = R^{-\frac{1}{12}}$

Tables III. and VIII. give the values of v for months and years.

Tables II. and IV. have been formed by the addition of the v 's.

Table I. has been formed by taking the reciprocals of the numbers in Table II., and are given exact to the next cent above the true value when the difference exceeds $\frac{1}{10}$ th of a cent. In all the other tables, the values are made true to the nearest decimal.

NOTE IV.—When the loan (P), instalment (A), and time are given, and it is required to find the rate of interest, R must be obtained from the equation

$$i = A \cdot \frac{1 - R^{-n}}{R - 1},$$

where n is the number of payments at the end of each interval, and $R = 1 + \text{interest per unit for one interval} = 1 + i$.

The solution of this equation, by Horner's method, though theoretically possible in all cases, becomes impracticable when n is not a small number, on account of the length of the operations, and in general it is better to proceed by Newton's approximation, which, however, converges but slowly and is often very troublesome to apply. Several approximate formulas have been devised to save this labor, of which the following may be noticed.

(i.) Halley's formula.

Calculating $\left(\frac{nA}{P}\right)^{\frac{2}{n+1}} - 1$, and calling this value β , then

$$i = \frac{6}{n-1} \left(1 - \sqrt{1 - \frac{n-1}{3} \beta} \right)$$

(ii.) Using the same notation,

$$i = \beta + \frac{1}{12} \frac{\beta^2}{n-1} + \frac{1}{72} \frac{\beta^3}{n-1^2}$$

(iii.) Baily's formula with the same notation :

$$i = \frac{(12 - n - 1\beta) \beta}{12 - 2n - 1\beta}$$

Of these formulas, (i.) fails altogether when β exceeds $\frac{3}{n-1}$ and becomes less correct the nearer β approaches to this limit; (ii.) is of more easy application than (i.), as the successive terms are readily found by aid of a table of squares and cubes; it becomes more and more erroneous under the same circumstances as (i.), the degree of error being indicated by the less convergence of the second and third terms relatively to the first term. (iii.) is also a modification of (i.); it fails when β gets beyond $\frac{6}{n-1}$ and becomes less correct as β approaches this limit. It is more troublesome to work than either (i.) or (ii.), but its great disadvantage is that it gives no indication in itself of its degree of error.

None of the three give good approximations when the period or the rate is large. For example, $A = 1$, $P = 9.99927484$, $n = 100$, $\beta = .04762$. Here (i.) fails; in (ii.) the terms diverge; (iii.) gives $i = .13477$, which is quite astray, the true value being $i = .10000$.

The method of "reversion of series" may also be used for the solution, but it is liable to the same objections as above unless the series is carried to so large a number of terms as to render the operation too laborious to be practically useful.

NOTE V.—The foregoing Tables will be found useful in enabling Building Societies to keep a proper account with each borrower. These accounts should be kept on the principle that whenever an instalment is paid, the borrower is charged with interest at the Society's rate on the sum in his possession since the date of the last payment, and credited with the amount paid; or, which comes to the same thing, instead of balancing an account whenever an instalment is paid, the borrower may at the end of the year be debited with the amount due at the beginning of the year, with interest on the same, and credited with the instalments paid in this interval with interest from the times of payment. This method, which is the one to be adopted in practice, is illustrated in the following examples. The multiplications that occur in the calculations are most conveniently performed by means of the Arithmometer of M. Thomas, of Colmar, France, an instrument which should be in the possession of every Building Society.* In these examples the books of the Society are closed on the first day of December in each year, payments made on that day being included in the accounts, and all loans are supposed to be contracted on the first day of a month.

* London agents: C. & J. Layton, 150 Fleet Street. A 12-figure machine will be found sufficient. In multiplying a sum by a factor, it will be better to begin from the left of the multiplier, as follows: Having set the multiplicand with the buttons, raise the plate and carry it to the right, so as to leave at least one hole to the left of the left-hand button. Multiply by the left hand figure of the multiplier; raise the plate and carry it one stage to the left; then multiply by the next figure, and so on. In this way the operation can be stopped without taking in unnecessary figures, when sufficient accuracy has been attained.

EXAMPLE 1.—August 1, 1872. A. B. borrows \$1,000.00, to be re-paid in 6 years by yearly instalments of \$231.30 each.

The amounts on the Dr. side are calculated by means of Table V. by multiplying the sum at the head of a column by the factor corresponding to the number of months between the dates at the head and foot of the column.

Thus in the first column 1,000 is multiplied by the factor 1.033062 corresponding to the interval 4 months, giving 1033.06.

The amounts on the Cr. side are calculated in like manner.

Thus in the second column the instalment, 231.30, is multiplied by the same factor, giving 238.95.

No instalment having been paid before Dec. 1, 1872, the borrower is charged with \$1,033.06 in opening the account for the next year. The difference between \$1,138.95 and \$238.95 is \$900.00, the present value of the mortgage on Dec. 1, 1873, and this is the amount with which the borrower is properly charged in opening the account for the next year. The account is closed Aug. 1, 1878, when there is a difference of nine cents between the Cr. and Dr. sides. This difference arises from the fact that the instalment, \$231.30, is somewhat greater than the true value. If the more accurate value, \$231.292, be taken, the account will be found to balance exactly. Omitting dates, &c., the account would then stand as follows:

Dr.

1,000.000	1,033.062	900.012	753.324	591.601	413.301	216.725
1,033.062	1,138.951	992.263	830.540	652.240	455.664	231.292

Cr.

0.000	231.292	231.292	231.292	231.292	231.292	231.292
0.000	238.939	238.939	238.939	238.939	238.939	231.292
1,033.062	900.012	753.324	591.601	413.301	216.725	0.000
1,033.062	1,138.951	992.263	830.540	652.240	455.664	231.292

EXAMPLE 2.—March 1, 1872. C. D. borrows \$1,000.00 to be repaid in 5 years by half-yearly instalments of \$129.51 each.

The amounts on the Dr. side are calculated as in Example 1.

The amounts on the Cr. side are found by multiplying each instalment by the factor corresponding to the proper interval, and adding together the products so obtained.

Thus for 1873 the first instalment is multiplied by the factor for 9 months, and the second instalment by the factor for 3 months.

EXAMPLE 3.—October 1, 1872. E. F. borrows \$1,000.00 to be repaid in 3 years by quarterly instalments of \$97.31 each.

The amounts on the Dr. side are calculated as in previous Examples.

The amounts on the Cr. side may be obtained by multiplying each instalment \$97.31 by the factor corresponding to the proper interval, and adding together the products so obtained.

Thus for 1873 the first instalment is multiplied by the factor for 11 months, the second by the factor for 8 months, the third by the factor for 5 months, and the fourth by the factor for 2 months, thus giving 410.52.

But this result is more easily obtained with the aid of Table VII by multiplying the instalment by the factor corresponding to the number of payments in Table VII, and then multiplying this result by the factor in Table V corresponding to the interval between the last payment and the end of the year.

Thus in the above example, the instalment 97.31 is multiplied by the factor 4.150624 in Table VII corresponding to four payments, and then by the factor 1.016597 in Table V, corresponding to the remaining period of 2 months, giving 410.52.

And in the last column for 1875 the amount of the instalments at date of last payment is given at once from Table VII by multiplying the instalment 97.31 by the factor corresponding to 4 payments, 4.150624, giving 403.90.

EXAMPLE 4.—September 1, 1872. G. H. borrows \$1,000.00 to be repaid in 3 years by monthly instalments of \$32.18 each.

The amounts on the Dr. side are obtained as before.

The amounts on the Cr. side are obtained by means of Table VI, by multiplying the instalment 32.18 by the factor corresponding to as many months as there have been instalments paid.

Thus in the first column 32.18 is multiplied by the factor 3.024562 corresponding to 3 months, giving 97.33. In the second column 32.18 is multiplied by the factor 12.553819 corresponding to 12 months, giving 403.98; and in the fourth column 32.18 is multiplied by the factor 9.299604 corresponding to 9 months, giving 299.26.

As in the previous examples, the small difference between the final amounts on the two sides 299.26 and 298.94 arises from the fact that the instalment was only given to the nearest cent above the true value; if the more correct value 32.17235 had been used, the balance would have come out exact, the final sums on each side of the account being 299.182. In the great majority of cases a small balance will be found to the advantage of the society, which may be carried to Profit and Loss.

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	1876.		1877.
P	Dec. 1 \$413.94		Dec. 1 \$216.65

Dr.		A. B.					
Present value of mortgage....	1872.	1872.	1873.	1874.	1875.	1876.	1877.
	Aug. 1....\$1,000.00	Dec. 1....\$1,033.06	Dec. 1....\$900.00	Dec. 1....\$758.80	Dec. 1....\$591.56	Dec. 1....\$413.24	Dec. 1....\$216.65
Amount to date ..	Dec. 1....\$1,033.06	1873. Dec. 1....\$1,138.95	1874. Dec. 1....\$929.25	1875. Dec. 1....\$830.51	1876. Dec. 1....\$652.19	1877. Dec. 1....\$455.60	1878. Aug. 1....\$231.21

Cr.		A. B.					
Instalment	1872. \$0.00	1873. Aug. 1....\$231.30	1874. Aug. 1....\$231.80	1875. Aug. 1....\$231.80	1876. Aug. 1....\$231.80	1877. Aug. 1....\$231.80	1878. Aug. 1....\$231.30
Amount to date ..	Dec. 1....\$0.00	Dec. 1....\$238.95	Dec. 1....\$238.95	Dec. 1....\$238.95	Dec. 1....\$238.95	Dec. 1....\$238.95	Aug. 1....\$231.30
Balance	1,033.06	900.00	753.30	591.56	413.24	216.65	0.00
	\$1,033.06	\$1,138.95	\$929.25	\$830.51	\$652.19	\$455.60	\$231.30

Dr.		C. D.					
Present value of mortgage....	1872.	1872.	1873.	1874.	1875.	1876.	
	Mar. 1....\$1,000.00	Dec. 1....\$943.22	Dec. 1....\$767.85	Dec. 1....\$574.50	Dec. 1....\$361.84	Dec. 1....\$126.53	
Amount to date ..	Dec. 1....\$1,075.93	1873. Dec. 1....\$1,039.90	1874. Dec. 1....\$846.55	1875. Dec. 1....\$633.39	1876. Dec. 1....\$398.88	1877. Mar. 1....\$129.45	

Cr.		C. D.					
Instalment	1872. Mar. 1....\$0.00 Sept. 1....129.15	1873. Mar. 1....\$129.51 Sept. 1....129.51	1874. Mar. 1....\$129.51 Sept. 1....129.51	1875. Mar. 1....\$129.51 Sept. 1....129.51	1876. Mar. 1....\$129.51 Sept. 1....129.51	1877. Mar. 1....\$129.51	
Amount to date ..	Dec. 1....\$132.71	Dec. 1....\$272.05	Dec. 1....\$272.05	Dec. 1....\$292.05	Dec. 1....\$272.05	Mar. 1....\$129.51	
Balance	943.22	767.85	574.50	361.84	126.53	0.00	
	\$1,075.93	\$1,039.90	\$846.55	\$633.39	\$398.88	\$129.51	

Dr.		E. F.			
Present value of mortgage....	1872.	1872.	1873.	1874.	
	Oct. 1....\$1,000.00	Dec. 1....\$1,016.40	Dec. 1....\$710.06	Dec. 1....\$372.82	
Amount to date ..	Dec. 1....\$1,016.40	1873. Dec. 1....\$1,120.58	1874. Dec. 1....\$782.84	1875. Oct. 1....\$403.86	

Cr.		E. F.			
Instalment	1872. \$0.00	1873. Jan. 1....\$97.31 Apr. 1....97.31 July 1....97.31 Oct. 1....97.31	1874. Jan. 1....\$97.31 Apr. 1....97.31 July 1....97.31 Oct. 1....97.31	1875. Jan. 1....\$97.31 Apr. 1....97.31 July 1....97.31 Oct. 1....97.31	
Amount to date ..	\$0.00	Dec. 1....\$410.52	Dec. 1....\$410.52	Oct. 1....\$403.90	
Balance	1,016.40	710.06	372.82	0.00	
	\$1,016.40	\$1,120.58	\$782.84	\$403.90	

Dr.		G. H.			
Present value of mortgage....	1872.	1872.	1873.	1874.	
	Sept. 1....\$1,000.00	Dec. 1....\$927.86	Dec. 1....\$618.43	Dec. 1....\$277.84	
Amount to date ..	Dec. 1....\$1,024.09	1873. Dec. 1....\$1,022.42	1874. Dec. 1....\$681.92	Oct. 1....\$298.94	

Cr.		G. H.			
Instalment	1872. Jan. 1.... Feb. 1.... Mar. 1.... Apr. 1.... May 1.... June 1.... July 1.... Aug. 1.... Sep. 1.... Oct. 1....\$32.18 Nov. 1....32.18 Dec. 1....32.18	1873. Jan. 1....\$32.18 Feb. 1....32.18 Mar. 1....32.18 Apr. 1....32.18 May 1....32.18 June 1....32.18 July 1....32.18 Aug. 1....32.18 Sep. 1....32.18 Oct. 1....32.18 Nov. 1....32.18 Dec. 1....32.18	1874. Jan. 1....\$32.18 Feb. 1....32.18 Mar. 1....32.18 Apr. 1....32.18 May 1....32.18 June 1....32.18 July 1....32.18 Aug. 1....32.18 Sep. 1....32.18 Oct. 1....32.18 Nov. 1....32.18 Dec. 1....32.18	1875. Jan. 1....\$32.18 Feb. 1....32.18 Mar. 1....32.18 Apr. 1....32.18 May 1....32.18 June 1....32.18 July 1....32.18 Aug. 1....32.18 Sep. 1....32.18 Oct. 1....32.18	
Amount to date ..	Dec. 1....\$97.88	\$403.98	\$403.98	\$299.26	
Balance	927.86	618.43	277.84	0.00	
	\$1,024.09	\$1,022.41	\$681.92	\$299.26	

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To find the present value of a sum due after a broken
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p. 10; s. (7); Tables III., VIII.

To find the amount, with interest, of a sum, after any
number of months, from 1 to 12. p. 8; s. (5); Table V.

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To find the present value of an instalment payable at the
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